# Mendham Township School District

## Mathematics Curriculum - 2012

## Honors Algebra

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 4: In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Critical Area 5: In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

| Units | Includes Standard Clusters* | Mathematical Practice Standards |
|---|---|---|
| **Unit 1**<br>Relationships Between Quantities and Reasoning with Equations | • Reason quantitatively and use units to solve problems.<br>• Interpret the structure of expressions.<br>• Create equations that describe numbers or relationships.<br>• Understand solving equations as a process of reasoning and explain the reasoning.<br>• Solve equations and inequalities in one variable. | **Make sense of problems and persevere in solving them.**<br><br>**Reason abstractly and quantitatively.**<br><br>**Construct viable arguments and critique the reasoning of others.**<br><br>**Model with mathematics.**<br><br>**Use appropriate tools strategically.**<br><br>**Attend to precision.**<br><br>**Look for and make use of structure.**<br><br>**Look for and express regularity in repeated reasoning.** |
| **Unit 2**<br>Linear and Exponential Relationships | • Extend the properties of exponents to rational exponents.<br>• Solve systems of equations.<br>• Represent and solve equations and inequalities graphically.<br>• Understand the concept of a function and use function notation.<br>• Interpret functions that arise in applications in terms of a context.<br>• Analyze functions using different representations.<br>• Build a function that models a relationship between two quantities.<br>• Build new functions from existing functions.<br>• Construct and compare linear, quadratic, and exponential models and solve problems.<br>• Interpret expressions for functions in terms of the situation they model. | |
| **Unit 3**<br>Descriptive Statistics | • Summarize, represent, and interpret data on a single count or measurement variable.<br>• Summarize, represent, and interpret data on two categorical and quantitative variables.<br>• Interpret linear models. | |
| **Unit 4**<br>Expressions and Equations | • Interpret the structure of expressions.<br>• Write expressions in equivalent forms to solve problems.<br>• Perform arithmetic operations on polynomials.<br>• Create equations that describe numbers or relationships.<br>• Solve equations and inequalities in one variable.<br>• Solve systems of equations. | |
| **Unit 5**<br>Quadratic Functions and Modeling | • Use properties of rational and irrational numbers.<br>• Interpret functions that arise in applications in terms of a context.<br>• Analyze functions using different representations.<br>• Build a function that models a relationship between two quantities.<br>• Build new functions from existing functions.<br>• Construct and compare linear, quadratic, and exponential models and solve problems. | |

The following chart details the Algebra curricular focus and is broken out into areas of content, skills, and concepts:

## Unit 1: Relationships Between Quantities and Reasoning with Equations

By the end of eighth grade students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. All of this work is grounded on understanding quantities and on relationships between them.

| UNIT ONE CCS | CCCS# | Comp & Content | Skills | Concepts |
|---|---|---|---|---|
| Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | NQ1 | Convert from one unit to another | *Choose and interpret units in formulas<br>*Label appropriately<br>*Choose and interpret the scale and origin in graphs and data displays | Reason quantitatively and use appropriate units to solve problems |
| Define appropriate quantities for the purpose of descriptive modeling. | NQ2 | | Define appropriate quantities for the purpose of descriptive modeling | |
| Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | NQ3 | | Choose the level of accuracy appropriate to limitations and measurement when reporting quantities (i.e. appropriate place to round and reasonableness of answers) | |
| Interpret expressions that represent a quantity in terms of its context. | ASSE1 | Review English to math vocab | *Interpret linear and exponential expressions<br>*Verbalize meaning of linear and exponential expressions | |

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| Interpret parts of an expression, such as terms, factors, and coefficients. | ASSE1A | Definitions of:<br>1. Term<br>2. Factor<br>3. Coefficient | | |
| Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.* | ASSE1B | | Interpret complicated expression including multi-step and multi-variable expressions including formulas | |
| Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* | ACED1 | *Linear function<br>*Exponential function | Write equations and inequalities in one variable and use them to solve problems | |
| Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | ACED2 | *The coordinate system<br>*Graphing points | *Write linear and exponential equations in two or more variables to solve problems<br>*Graph linear and exponential equations in the coordinate plane | |
| Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | ACED3 | Definition of constraints | *Graph systems of equations and inequalities in the coordinate plane<br>*Represent constraints by equations or inequalities, and by systems of equations and/or inequalities | Interpreting solutions as viable or non-viable in the context of a problem |
| Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* | ACED4 | | *Transform formulas using inverse operations | Use the transformed formula to solve multiple problems for the solved variable |

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| Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | AREI1 | *Inverse operations<br>*Properties of equality | Verbalize the steps in solving a linear equation | |
| Solve linear equations and inequalities in one variable, including<br>equations with coefficients represented by letters. | AREI3 | | Solve linear equations and inequalities in one variable | Solve literal equations |

## Unit 2: Linear and Exponential Relationships

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

| UNIT TWO CCS | CCCS# | Comp & Content | Skills | Concepts |
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| Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.* | NRN1 | *Exponents<br>*Rational exponents<br>*Radicals | Extend properties of exponents to rational exponents | |

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| Rewrite expressions involving radicals and rational exponents using the properties of exponents. | NRN2 | | Rewrite expressions involving radicals and rational exponents using the properties of exponents | |
| Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | AREI5 | System of linear equations | Solve a system of equations by using linear combination/elimination | |
| Solve systems of linear equations exactly and approximately(e.g., with graphs), focusing on pairs of linear equations in two variables. | AREI6 | *Parallel lines = no solution<br>*Coinciding lines = infinite number of solutions | Solve a system of equations by graphing and the substitution method | |
| Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | AREI10 | Linear and exponential graphs | Use a table of values to create the graph of two-variable linear and exponential equations | |
| Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | AREI11 | Solution to a system of equations is the point of intersection of their graphs | Find the solution to a system of linear or exponential equations by using technology to graph and to make a table of values | |
| Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | AREI12 | *Boundary line<br>*Inequality signs and how they affect placement of shading | *Graph solutions to linear inequality in two variables<br>*Graph solutions to systems of linear inequalities in two variables as intersection of half planes | Construct meaning for the solution to an inequality by graphing |

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| Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*. | FIF1 | *Domain<br>*Range<br>*Relation<br>*Function | Use the vertical line test and mapping diagram to identify a function from a relation | Understand the concept of a function as a one-to-one correspondence |
| Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | FIF2 | Function notation | *Evaluate functions given a domain<br>*Interpret function notation in context | |
| Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.* | FIF3 | *Arithmetic sequence<br>*Geometric sequence<br>*Recursive patterns | Write a function rule to model a sequence | |
| For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* | FIF4 | *Intercepts, intervals where a function is increasing, decreasing, positive, or negative<br>*Parent linear and exponential function | *Sketch a graph of a linear or exponential function showing key features given a verbal description<br>*Identify key features of a graph given its function rule | |
| Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* | FIF5 | *Discrete data<br>*Continuous data | Relate domain of a function to its graph (ie interpret domain values in context) | |

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| Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | FIF6 | *Understand rate of change/slope<br>*Slope formula | *Calculate average rate of change from a graph or from data represented symbolically<br>*Estimate rate of change from a graph | |
| Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | FIF7 | | Graph functions expressed symbolically by hand and using technology | |
| Graph linear and quadratic functions and show intercepts, maxima, and minima. | FIF7A | Intercepts | Graph linear functions | |
| Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | FIF7E | Intercepts | Graph exponential functions | |
| Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | FIF9 | | | Compare properties of two functions represented in different ways (algebraically, graphically, in tables, or verbally) |
| Write a function that describes a relationship between two quantities. | FBF1 | | Write a function rule to model a relationship between quantities | |
| Determine an explicit expression, a recursive process, or steps for calculation from a context. | FBF1A | *Explicit expressions<br>*Recursive process | *Determine an explicit expression, a recursive process, or steps for calculation from a context (ie determine whether a linear or exponential model) | |

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| Combine standard function types using arithmetic operations.<br>*For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* | FBF1B | | Transform a linear/exponential function by adding, subtracting, or multiplying by a constant number | |
| Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | FBF2 | | *Write arithmetic and geometric sequences with an explicit formula<br>*Write a function rule to model arithmetic and geometric sequences | |
| Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with<br>cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | FBF3 | | *Build new functions from existing parent (linear and exponential) functions<br>*Use technology to show how changes to the parent function affect the graph<br>*Relate a vertical translation of a linear function to its y-intercept | |
| Distinguish between situations that can be modeled with linear functions and with exponential functions. | FLE1 | | | Distinguish between situations that can be modeled with a linear function and exponential function |
| Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals. | FLE1A | *Equal differences (aka common difference)<br>*Equal factors (aka common ratio) | | Prove linear functions grow by equal differences and exponential functions grow by equal factors |
| Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | FLE1B | | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another | |

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| Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | FLE1C | Percent rate:<br>*Decay factor <1<br>*Growth factor >1 | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval | |
| Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | FLE2 | | Construct linear and exponential functions given a graph, a description of a relationship, or two input-output pairs | |
| Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | FLE3 | | Compare linear growth to exponential growth by observing graphs and tables | |
| Interpret the parameters in a linear or exponential function in terms of a context. | FLE5 | | Interpret the parameters in a linear or exponential function in context (ie do negatives make sense? Fractions?) | |

## Unit 3: Descriptive Statistics

Experience with descriptive statistics began as early as Grade 6. Students were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatterplots and recognizing linear trends in data. This unit builds upon that prior experience, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

| UNIT THREE CCS | CCCS# | Comp & Content | Skills | Concepts |
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| Represent data with plots on the real number line (dot plots, histograms, and box plots). | SID1 | *Dot plots<br>*Histogram<br>*Box plots | Represent data with plots on the real number line appropriately | |

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| Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | SID2 | *Shape<br>*Median<br>*Mean<br>*IQR<br>*Standard deviation | *Compare two or more data sets<br>*Compute median, mean, IQR, standard deviation | |
| Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | SID3 | Outliers | Determine algebraically any outliers | Summarize data differences and effects on outliers |
| Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | SID5 | *Categorical data<br>*Quantitative variables | *Summarize categorical data in frequency tables<br>*Interpret relative frequencies in context<br>*Recognize trends in data | |
| Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | SID6 | | Represent data in a scatter plot and describe how the variables are related | |
| Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models* | SID6A | Trend line | *Fit a function to the data<br>*Draw a trend line and solve problems in context of the data | |
| Informally assess the fit of a function by plotting and analyzing residuals. | SID6B | | Describe data relative to the trend line | |
| Fit a linear function for a scatter plot that suggests a linear association. | SID6C | | Fit a linear function for a scatter plot (ie write an equation for the trend line) | |
| Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | SID7 | | | Interpret slope (rate of change) and the intercept (constant term) of a linear model in context |

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| Compute (using technology) and interpret the correlation coefficient of a linear fit. | SID8 | Correlation coefficient | Use technology to compute line of best fit and the correlation coefficient | |
| Distinguish between correlation and causation | SID9 | Causation vs. correlation | | |

## Unit 4: Expressions and Equations

In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

| UNIT FOUR CCS | CCCS# | Comp & Content | Skills | Concepts |
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| Interpret expressions that represent a quantity in terms of its context.★ | ASSE1 | English to math vocabulary | Interpret and verbalize quadratic and exponential expressions in context | |
| Interpret parts of an expression, such as terms, factors, and coefficients | ASSE1A | Definitions of:<br>1. Term<br>2. Factor<br>3. Coefficient | | |
| Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.* | ASSE1B | | Interpret complicated expressions including multi-step, multi-variable expressions and formulas | |
| Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).* | ASSE2 | | Write equivalent expressions by simplifying or by representing expressions in different forms (ie recognizing different forms of equivalent expressions) | |
| Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | ASSE3 | Properties of operations | *Write equivalent forms of expressions<br>*Explain properties of the quantity represented by the expression | |

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| Factor a quadratic expression to reveal the zeros of the function it defines. | ASSE3A | Identify zeroes of a quadratic function (by graphing) | Factor a quadratic expression to reveal the zeroes of a quadratic function (ie solve a quadratic equation by factoring) | |
| Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | ASSE3B | Identify maximum or minimum value of a quadratic function (ie vertex) by graphing | Solve quadratic equations by completing the square (to reveal the max/min value of the function—vertex) | Derive the Quadratic Formula by completing the square |
| Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* | ASSE3C | Properties of exponents | Transform exponential functions by using properties of exponents | Exponential growth and decay |
| Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | AAPR1 | Definitions of:<br>1. Polynomial<br>2. Degree of a polynomial<br>3. Types of polynomials<br>4. Closure Property | *Add, subtract, multiply polynomials<br>*Classify polynomial according to name and degree | |
| Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* | ACED1 | *Quadratic functions<br>*Simple rational Functions | Write equations and inequalities in one variable (linear, quadratic, simple rational, and exponential) and use them to solve problems | |

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| Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales | ACED2 | *The coordinate system<br>*Graphing points | *Write linear and exponential equations in two or more variables (linear, quadratic, simple rational, and exponential) to solve problems<br>*Graph linear, quadratic, simple rational, and exponential equations in the coordinate plane | |
| Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* | ACED4 | | *Transform formulas involving squared variables using inverse operations | Use the transformed formula to solve multiple problems for the solved variable |
| Solve quadratic equations in one variable. | AREI4 | | Solve quadratic equations in one variable | |
| Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form *(x – p)2 = q* that has the same solutions. Derive the quadratic formula from this form. | AREI4A | | Use completing the square to transform a quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions | Derive the Quadratic Formula |
| Solve quadratic equations by inspection (e.g., for *x2* = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*. | AREI4B | Existence of complex number system (but do not solve quadratic equations with complex solutions) | *Solve quadratic equations by:<br>1. Inspection<br>2. Square roots<br>3. Completing the square<br>4. Factoring<br>5. The Quadratic Formula<br>*Recognize when a quadratic equation gives complex solutions and write them as a +/- bi for real numbers a and b | |

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| Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.* | AREI7 | | Solve simple systems of linear equations and quadratic equations graphically and algebraically (including systems containing one linear and one quadratic equation) | |

**Unit 5: Quadratic Functions and Modeling**

In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

| UNIT FIVE CCS | CCCS# | Comp & Content | Skills | Concepts |
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| Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | NRN3 | Definitions of :<br>1. Closure Property<br>2. Rational numbers<br>3. Irrational numbers | | Effects on outcome of addition and multiplication of rational and irrational number will result in an irrational number |
| For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities,<br>and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* | FIF4 | *Intercepts, intervals where a function is increasing, decreasing, positive, or negative, relative maximums and minimums, symmetries, end behavior, and periodicity<br>*Parent quadratic function | *Sketch a graph of a quadratic function showing key features given a verbal description<br>*Identify key features of a graph given its function rule | |

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| Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* | FIF5 | *Discrete data<br>*Continuous data | Relate domain of a function to its graph (ie interpret domain values in context) | |
| Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | FIF6 | *Understand rate of change/slope<br>*Slope formula | *Calculate average rate of change from a graph or from data represented symbolically<br>*Estimate rate of change from a graph | |
| Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | FIF7 | | Graph functions expressed symbolically by hand and using technology | |
| Graph linear and quadratic functions and show intercepts, maxima, and minima. | FIF7A | Intercepts | Graph quadratic functions and show intercepts, maxima, and minima | |
| Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | FIF7B | *Square root<br>*Cube root<br>*Piecewise defined functions | Graph square root, cube root, piecewise defined functions, including step functions and absolute value functions | |
| Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | FIF8 | | Write a function in equivalent forms | |
| Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | FIF8A | *Zeroes<br>*Extreme Values<br>*Symmetry<br>*Axis of symmetry | *Factor and complete the square in a quadratic function to show zeroes, extreme values, and symmetry and interpret in context | |

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| Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.* | FIF8B | | Classify an exponential function as exponential growth or exponential decay | Interpret expressions for exponential functions |
| Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | FIF9 | | Compare two quadratic functions when represented in different forms (algebraically, graphically, numerically in tables, or verbal descriptions) | |
| Write a function that describes a relationship between two quantities. | FBF1 | | Write a quadratic function to model a relationship between quantities | |
| Determine an explicit expression, a recursive process, or steps for calculation from a context. | FBF1A | *Explicit expressions<br>*Recursive process | *Determine an explicit expression, a recursive process, or steps for calculation from a context (ie determine if a quadratic model) | |
| Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a* | FBF1B | | Transform a quadratic function by adding, subtracting, or multiplying by a constant number | |
| Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | FBF3 | | *Build new functions from existing parent (quadratic) functions<br>*Use technology to show how changes to the parent function affect the graph | |

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| Find inverse functions. | FBF4 | *Inverse function | Find inverse functions | |
| Solve an equation of the form *f(x) = c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) = 2 x3 or f(x) = (x+1)/(x-1) for* $x \neq 1$. | FBF4A | | *Solve an equation of form f(x) = c for a simple function f that has an inverse<br>*Write an expression for inverse | |
| Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | FLE3 | | Compare linear growth and exponential growth to quadratic by observing graphs and tables | |